

Thermodynamic optimization of Stirling heat pump based on multiple criteria



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ABSTRACT

In this research work, a connectionist investigation of irreversible Stirling heat pump cycles that includes both internal and external irreversibilities together finite heat capacities of external reservoirs was carried out. Finite temperature difference between the external fluids and the working fluids through the heat sink and heat source causes an external irreversibility. On the other hand, regenerative heat loss and entropy generation through the cycle are the main source of the internal irreversibilities generation. Three objective functions including the heating load (R_H) and coefficient of performance (COP) have been considered simultaneously maximized, on the other hand at the same time the input power of the Stirling heat pump is minimized. To assess this idea, Multi-objective optimization approach based on NSGA-II method has been utilized which following variables have been considered as decision variables such as 1 – the effectiveness of the hot-side heat exchanger, 2 – the performance of the cold-side heat exchanger, 3 – the rate of heat capacitance through the heat sink, temperature ratio (T_c/T_h), 4 – rate of the heat capacitance through the heat source and 5-temperature of cold side. By applying addressed multi-objective optimization approach, Pareto optimal frontier determined and utilizing different decision-making techniques that includes the LINMAP, TOPSIS and fuzzy Bellman–Zadeh approaches help us to figure out a final optimal solution.

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1. Introduction

Throughout cycle approaches of refrigerators, air conditioning and heat pump (RAC & HP) systems, Stirling and Ericsson cycles play vital role which have been used by a different experts in the approach of realistic frameworks to determine the generation of desired temperatures. Recently huge amounts of attentions have been made by various Scientists on the addressed cycles. Curzon and Ahlborn were introduced the concept of finite time thermodynamics [1] by performing a new work on Carnot heat engine whereas they highlighted the effectiveness of addressed engine

that operates at maximum power is calculated with their introduced formula $\left[\eta_m = 1 - \sqrt{\frac{T_c}{T_h}}\right]$. It should be noted that the output of their proposed formula is always lower than the routine formula of Carnot $\left[\eta_c = 1 - \frac{T_c}{T_h}\right]$ while closed more appropriate with the actual efficiencies of operating systems given by them. In addition the same theory on the finite time Carnot heat engine with limited external reservoirs heat capacities was also performed by Wu [2]. Based on his work, the inlet temperatures of the external fluids are the most important variables in the highest power output and the performance at highest power. Due to loose of “Natural Maximum” in the reversed Carnot cycle, the simple C–A are calculations unable to perform as noted previously by Leff and Teeters [3]. Determination of the COP of endoreversible Carnot heat pump that operates at lowest power input for a desired heating load by implementing the Lagrangian approach of indefinite factor was conducted by Blankard [4]. Wu has been effectively implemented the theorem of finite time thermodynamics to indicate the optimal

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Nomenclature

C	rate of heat capacitance (W K^{-1})
C_v	specific heat capacity ($\text{J mol}^{-1} \text{K}^{-1}$)
Cl_i	decision index in TOPSIS method (–)
COP	the coefficient of performance (W m^{-2})
i	i th objective (–)
j	j th solution (–)
n	number of mole (–)
R_H	heating load (W)
P	Output power (W)
Q	heat (J)
R	the gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)
S	entropy (J K^{-1})
T	temperature (K)
t	time (s)
X	vector of decision variables (–)
x	temperature ratio (–)
W	output work (J)

Subscripts

1, 2, 3, 4	state points
c	cold side
dis	distance
H	heat source
h	hot side
L	cold side/heat sink
R	regenerator

Greek

η	thermal efficiency (–)
ε	effectiveness (–)
ϕ	internal irreversibility parameter (–)
λ	the volume ratio (–)
α	proportionality constant (J K^{-1})

performance of the various cycles such as endo-reversible Carnot, Stirling RAC & HP and Brayton systems for various situations [5,6]. Nova days, optimum performance parameters of the Stirling refrigerators and Stirling heat pumps have been investigated systematically by performing finite time thermodynamics as a function of operating conditions [7–17]. Chen et al. developed mathematical and thermodynamic models for the coefficient of performance, heating load and cooling load for heat pumps and refrigerators with finite heat transfer efficiency [7–12]. Kaushik et al and Feidt et al investigated the impacts of irreversibilities in heat pump cycles [13–17].

To consider all above-mentioned declares, in this research three objective functions counting input power, the coefficient of performance (COP) and heating load (R_H) have been studied. Furthermore, the multi-objective optimization method was performed with Six decision variables that includes rates of the heat capacitance through heat sink and heat source, the cold-side and hot-and heat exchangers performance's, ratio of temperatures ($\frac{T_H}{T_C}$) and temperature of sink side (T_C).

Multi-objective optimization is useful way for solving engineering problems at various fields for research [18–20].

Solving the optimization of multi-objective issues is a severely hard target and needs simultaneous satisfaction of various and often contradicting objectives. Owing to this fact, evolutionary algorithms (EAs) have been established and developed through the 18th century in an endeavor to randomly solve multi-objective issues [21]. An acceptable route to a multi-objective quandary is to inquiry an assortment of routes, each of them convinces the objectives at a satisfactory degree away being overcome by another route [22]. Multi-objective optimization quandaries universally represent a feasibly innumerable group of ways which called Pareto frontier, whose assessed vectors show the foremost feasible exchanges throughout the objective function area. By the way, multi-objective optimization of energy systems and diverse thermodynamic have been applied in different field of science these days [23–31].

Throughout this communication, multi-objective optimization algorithms has been utilized to maximize the coefficient of performance (COP_H) and heating load (R_H) and minimize the input power. To assess this valuable goal, three robust, cheap and fast decision making approaches have been used to determine the optimum solutions through the multi-objective optimization process. Moreover, error analysis has been carried out indicate statistical analysis of the gained solutions of various decision making techniques.

More details about the mentioned decision making that includes LINMAP, TOPSIS and Fuzzy methods are demonstrated in the following sections.

2. System description

Gas or a magnetic material can be used as a working fluids of the Stirling cycle (Nickel-based super alloys are the primary materials used in the Stirling-cycle portion of the Engine). Also, different working fluids lead to different performance characteristic of Stirling systems.

As depicted in Fig. 1, the cycle comprises two isochoric and two isothermal processes when the addressed working fluids of the cycle are the ideal gas. The mentioned cycle proximate the expansion stroke of an actual cycle as an isothermal process 1–2 while an irreversible isothermal heat added from a heat source of finite heat capacity at temperature T_c that the temperature changes in range of T_{L1} to T_{L2} .

Addition of the heat from the regenerator to the addressed working fluid was considered as an isochoric process 2–3. The compression stroke was considered as isothermal processes 3–4 with an irreversible heat rejection to the heat sink of finite heat capacity at temperature T_h that temperature changes in range of T_{H1} to T_{H2} . Lastly, rejection of the heat from the working fluid that comes to the regenerator was considered as an isochoric process 4–1. As expressed previously, the heat transfer processes 1–2 and 3–4 in an actual cycle must happen throughout finite time. This needs the addressed heat processes would be preferred in the finite temperature difference and consequently, specified as stand hardly irreversible.

In addition, the change of entropy through process 3–4 is greater than the change of entropy through process 1–2. So, for each cycle, there are some net entropy generation as well as internal irreversibility parameter (ϕ); which defined as the rate of entropy change throughout the expansion process to the rate entropy change in the compression process. Furthermore, some heat losses entire the regenerator, as a perfect regeneration needs unlimited regeneration area or time. Therefore, it is necessary to study an actual regenerator. Involving all the above mentioned aspects, the addressed heat pumps approaches turn irreversible. As mentioned previously, the external irreversibility is caused by difference of limited temperature between the external reservoirs and the cycle and on the other hand, regenerative loss and entropy generation results internal irreversibilities.

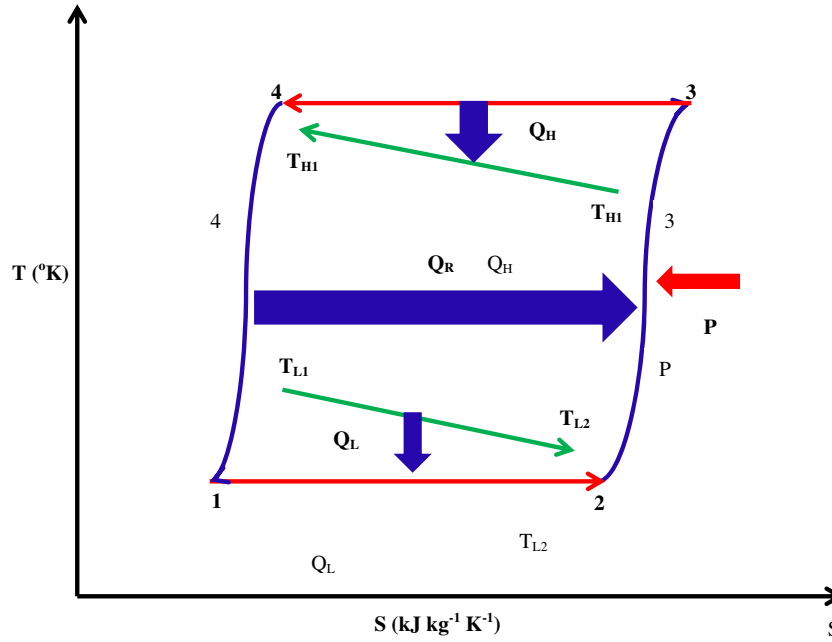


Fig. 1. Carton of the temperature-entropy diagram for an irreversible Stirling heat pump cycles [16,17].

3. Thermodynamic analysis

By defining Q_c and Q_h that represent the value of heat gained at temperature T_c from the source and the heat deliver at temperature T_h to the sink, respectively, then [8,9]:

$$Q_h = T_h \Delta S = C_H(T_{H2} - T_{H1})t_H = U_H A_H (LMTD)_H t_H \quad (1)$$

$$Q_c = T_c \Delta S = C_L(T_{L1} - T_{L2})t_L = U_L A_L (LMTD)_L t_L \quad (2)$$

where entropy change is $\Delta S = nR \ln \lambda$, and λ represents the volume ratio and n denotes the mole numbers of the working fluid. t_L ; t_H and C_L ; C_H represent the times of heat addition/rejection and rates of heat capacitance in source/sink reservoirs, $H1$ and $L1$ are related to cold-side and hot-side, correspondingly. $U_L A_L$ and $U_H A_H$ indicate the products of the overall heat transfer coefficient and area and $(LMTD)_L$ and $(LMTD)_H$ denote the log mean temperature difference on source- and sink -side, correspondingly, that formulated as:

$$(LMTD)_H = \frac{(T_h - T_{H1}) - (T_h - T_{H2})}{\ln \left(\frac{T_h - T_{H1}}{T_h - T_{H2}} \right)} \quad (3)$$

$$(LMTD)_L = \frac{(T_{L1} - T_c) - (T_{L2} - T_c)}{\ln \left(\frac{T_{L1} - T_c}{T_{L2} - T_c} \right)} \quad (4)$$

Generally, these cycles do not states the perfect regeneration situation, hence it is acceptable to involve the regenerator loss that is proportionate to difference of the temperature between the two mentioned isothermal processes as supposed by previous researches in Refs. [13–17],

$$\Delta Q_R = nC_v(1 - \varepsilon_R)(T_h - T_c) \quad (5)$$

n stands for the number of moles through the working fluid, C_v stands for the working fluid specific heat capacity's throughout the regenerative processes, ε_R denotes the performance of regenerator.

Dedicated to the impact of finite heat transfer irreversibility's, the time of regenerative would be finite, same as two referred isothermal processes [13–17], and become

$$t_R = t_3 + t_4 = 2\alpha(T_h - T_c) \quad (6)$$

In which α represents the constant of proportionality that is not depend on the temperature difference however; changes with the characteristics of the regenerative material. Therefore, the total cycle time t_{cycle} can be determined by following formula:

$$t_{cycle} = (t_H + t_L + t_R) \quad (7)$$

By considering all the above mentioned irreversibilities through calculation, the value of net heat that gained from the source and amount of heat deliver to the sink could be determine by

$$Q_H = Q_h - \Delta Q_R \quad (8)$$

$$Q_L = Q_c - \Delta Q_R \quad (9)$$

Fundamental parameters of any heat pump are power input, heating load and the COP_H . Applying the above equations, obtained by the power input expressions, heating load and heating COP_H are:

$$P = \frac{(Q_H - Q_L)}{t_{cycle}} \quad (10)$$

$$R_H = \frac{Q_H}{t_{cycle}} \quad (11)$$

$$COP_H = \frac{R_H}{P} \quad (12)$$

In addition, for an irreversible cycle the thermodynamic second law's provides

$$\frac{Q_c}{T_c} - \frac{Q_h}{T_h} < 0 \quad (13)$$

Or

$$\frac{Q_c}{T_c} = \phi \frac{Q_h}{T_h} \quad (14)$$

In which ϕ represents the irreversibility parameter that is usually lower than unity for an actual cycle. Therefore, by considering Eqs. (6)–(14) following expressions could be develop:

$$P = \frac{x - \phi}{\left(\frac{x}{\varepsilon_H C_H (x T_c - T_{H1})} \right) + \left(\frac{\phi}{\varepsilon_L C_L (T_{L1} - T_c)} \right) + b_1(x - 1)} \quad (15)$$

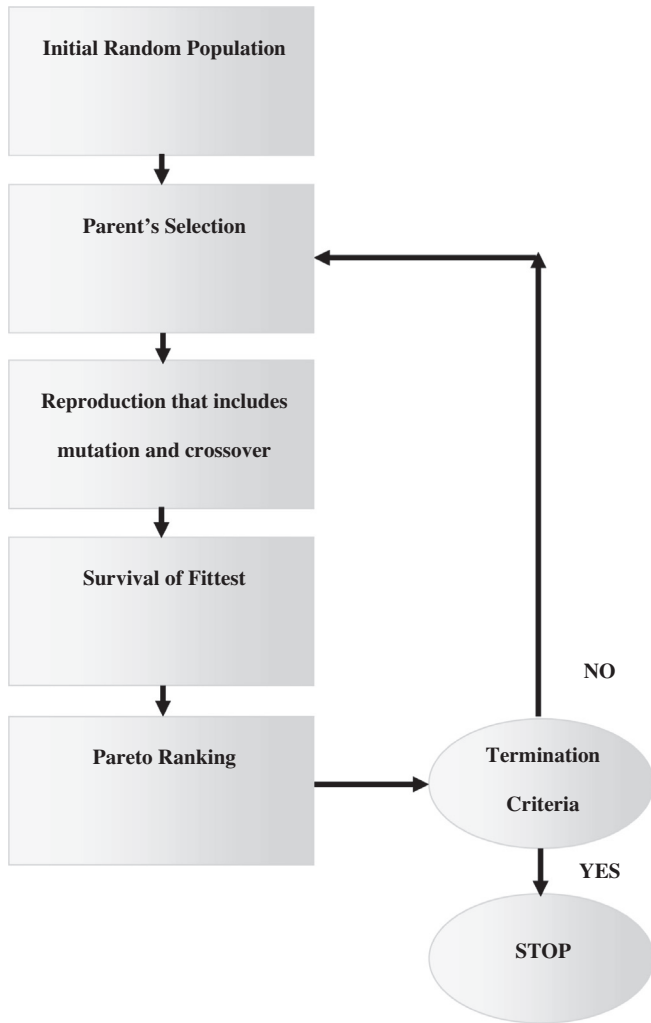


Fig. 2. Box-Chart of the executed multi-objective optimization approach in the current research work [23–27,31,32,34].

$$R_H = \frac{x - F_1(x - 1)}{\left(\frac{x}{\varepsilon_H C_H (x T_c - T_{H1})}\right) + \left(\frac{\phi}{\varepsilon_L C_L (T_{L1} - T_c)}\right) + b_1(x - 1)} \quad (16)$$

$$COP_H = \frac{x - F_1(x - 1)}{x - \phi} \quad (17)$$

$$x = \frac{T_h}{T_c} \quad (18)$$

$$b_1 = \frac{2\alpha}{nR \ln \lambda} \quad (19)$$

$$F_1 = \frac{C_v(1 - \varepsilon_R)}{R \ln \lambda} \quad (20)$$

4. Multi-objective optimization

As noted previously, the multi-objective optimization approach evolved from evolutionary procedure was employed to optimize Stirling heat pump system to figure out the prior parameters of system design. To evaluate this, Genetic algorithms (GAs) perform stochastic and iterative search approach to figure out optimal way and follow in a facilitated behavior principle of biological evolution. The general concept of Genetic algorithms is displayed in Fig. 2 [21–24,31,33].

The system's input power (P), the heating load (R_H), and the coefficient of performance (COP) signified by Eqs. (15)–(17) were considered as three objective functions for this study, correspondingly.

Throughout this communication following variables have been considered as decision variables:

T_c is the cold side temperature's, x the temperature ratio $\left(\frac{T_h}{T_c}\right)$, C_H the rate of heat capacitance in the heat source, C_L the rate of heat capacitance in the heat sink, ε_L the cold-side heat exchanger effectiveness's, ε_H the hot-side heat exchanger effectiveness's.

It is worth bearing in mind that, to reach better solution in optimization process while the decision variables may be changed, the addressed variables are normally required to fall in a reasonable interval. Throughout this research work, following limits have been considered to solve objective functions:

$$0.5 \leq \varepsilon_H \leq 0.8 \quad (21)$$

$$0.5 \leq \varepsilon_L \leq 0.8 \quad (22)$$

$$235 \leq T_c \leq 240K \quad (23)$$

$$600 \leq C_H \leq 1000 \quad (24)$$

$$600 \leq C_L \leq 1000 \quad (25)$$

$$1.3 \leq x \leq 1.4 \quad (26)$$

5. Decision-making through the multi-objective optimization process

Decision-making for collecting of the better optimal route from available routes through the multi-objective optimization approach has a vital role. There are a number of techniques for decision-making throughout the decision issues. These techniques could be utilized for choosing a better optimum route from the Pareto frontier. Meanwhile, dimension of the objectives throughout the multi-objective optimization issue may be varied. Thus, unification of the scales and dimensions of the objective space should be applied before making a decision. Put it in other way, vectors of objectives would be non-dimensionalized before the decision-making process. There are various approaches of non-dimensionalization employed throughout the decision making techniques that includes Euclidean non-dimensionalization, and fuzzy non-dimensionalization [31,34].

6. Non-dimensionalization methods

6.1. Euclidean non-dimensionalization

F_{ij}^n represents the matrix of objectives for different solutions of the Pareto frontier in which i is denotes the index of each route on the Pareto frontier, and j stands for the index of the objective throughout objective area. In this method, a non-dimensionalized objective, F_{ij}^n , expressed as follow [31,34],

$$F_{ij}^n = \frac{F_{ij}}{\sqrt{\sum_{i=1}^m (F_{ij})^2}} \text{ for minimization and maximization goals} \quad (27)$$

6.2. Fuzzy non-dimensionalization

Throughout this approach, a non-dimensionalized objective, F_{ij}^n , is formulated as [31,34],

$$F_{ij}^n = \frac{F_{ij} - \min(F_{ij})}{\max(F_{ij}) - \min(F_{ij})} \text{ for maximization objectives} \quad (28a)$$

$$F_{ij}^n = \frac{\max(F_{ij}) - F_{ij}}{\max(F_{ij}) - \min(F_{ij})} \text{ for minimization objectives} \quad (28b)$$

Throughout this study most legendary and standard kind of decision-making processes such as the TOPSIS, LINMAP and fuzzy Bellman–Zadeh approaches are employed simultaneously while to gain the optimal solution, the three addressed methods have been utilized. The Bellman–Zadeh approach conducts the fuzzy non-dimensionalization whereas; the two rest approaches utilize Euclidean non-dimensionalization. The further parts are explained these decision-making procedures [31,34].

7. Decision making methods

7.1. Bellman–Zadeh decision-making method

Throughout the fuzzy method that introduced by Bellman and Zadeh, the intercross point of entire fuzzy index and limits is defined as the final decision meanwhile is introduced by its relevant membership function. Through this approach, a membership function's matrix was introduced while the vertical line of the addressed matrix contains the objective fuzzy membership function. On the other hand, the membership matrix row's represent values of the membership functions for gained route from the Pareto frontier. Thus, the quantity of objectives is the same as quantity of vertical lines through the membership function matrix. Also, the quantity of solutions located on the Pareto frontier is the same as the quantity of horizontal lines throughout the referred matrix. Detail for method of definition for the membership function is described in Refs. [23,24,31,34]. The fuzzy membership function for each route is an assortment at the lowest magnitude of all the objectives membership functions for the suggested route. So, a membership vector's is gained that its elements are in the lowest membership objective functions at each route. Lastly, the greatest value of these minimum is collected as a prior route from

Pareto frontier. To put it in other way, through the decision making by the Bellman–Zadeh fuzzy approach, a highest value of the lowest membership function of objectives is chosen as the prior optimal route. For more details refer to Refs. [23,24,31,34].

7.2. LINMAP decision-making approach

In the LINMAP approach a definition that called “ideal point” should be defined. Ideal point is introduced as the point on the Pareto frontier that each objective is optimized irrespective to the considering other objectives. It should be noted that, the optimum value of each objectives through multi-objective optimization and single-objective optimization are impossibility equal. Thus, the addressed ideal data point is not placed on the Pareto frontier. Throughout the LINMAP approach, after Euclidean process on the all of non-dimensionalization objectives, the deviation of each route on the Pareto frontier from the introduced ideal point in space is indicated. Lastly, a route with a lowest distance in space from an ideal data point is chosen as a prior optimal route. To gain more description about the LINMAP approach please refer to the following Refs. [23,24,34].

7.3. TOPSIS decision-making approach

Through this approach, furthermore a “non-ideal point” is also introduced. The non-ideal data point defined as the latitude throughout the objective spatial that each objective includes its worst magnitude. Thus, another criterion for final solution selection is referred to distance in space from the non-ideal however; the distance in space from the ideal solution also considered as another criteria. Owing to these criteria, final route in the TOPSIS approach is selected based on the highest deviation from the non-ideal point and lowest deviation from the ideal point simultaneously. To gain more explanation of the TOPSIS approach following literatures should be read [34].

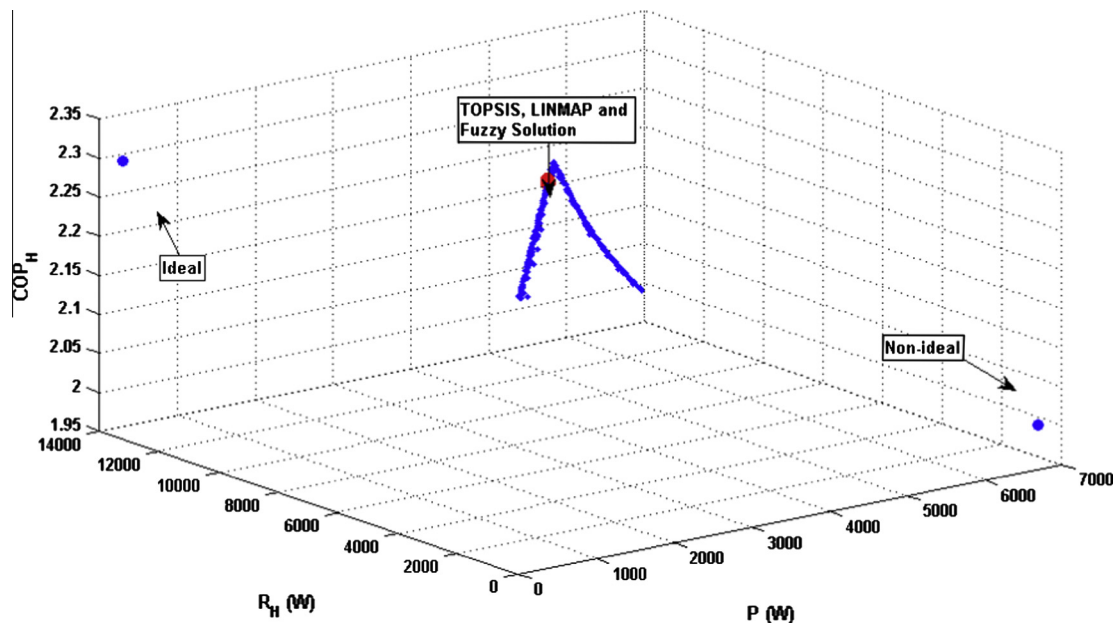


Fig. 3. Optimum of Pareto frontier throughout the space of objectives.

8. Result and discussion

As noted previously, by utilizing the multi-objective optimization approach that proceeds according to the NSGA-II procedure, maximization of the heating load (R_H), and coefficient of performance (COP) and minimization of the input power are simultaneously.

Here, optimization has been conducted with objective functions which are presented by Eqs. (15)–(17) restrictions that are proposed with Eqs. (21)–(26).

Design parameters through optimization process are the temperature ratio ($\frac{T_h}{T_c}$), the temperature of cold side, the rate of heat capacitance through the heat source, the rate of heat capacitance through the heat sink, effectiveness of the cold-side heat exchanger, the hot-side heat exchanger effectiveness's.

With the aim of previous researches, following Stirling heat pump specifications have been involved [9],

$T_{H1} = 330$ K, $T_{L1} = 290$ K, $\varepsilon_R = 0.9$, $\lambda = 2$, $n = 1$, $C_v = 15$ (J mol⁻¹ K⁻¹), $R = 4.3$ (J mol⁻¹ K⁻¹), $\phi = 0.8$, $\alpha = 10^{-5}$ (s K⁻¹).

Pareto optimal frontier for the three addressed objective functions that include input power, heating load and the coefficient of performance are represented in Fig 3. The selected points due to the decision making approaches are depicted, too. As shown in Fig 3. The three method of decision making referred to same point.

Pareto optimal frontier for input power and heating load are depicted in Fig 4, which are two objective functions. The collected points owing to the decision making techniques are illustrated, too. It can be seen from Fig 4 that the optimal points from the three methods of decision making are approached. It was shown that optimal value of P was 3000–3500 W and optimal value of R_H was 6000–8000 W.

Pareto optimal frontiers for input power and the coefficient of performance are shown in Fig 5 that include two mentioned OFs. The depicted points owing to the decision making techniques are demonstrated, too. It is seen that optimal value of P varied

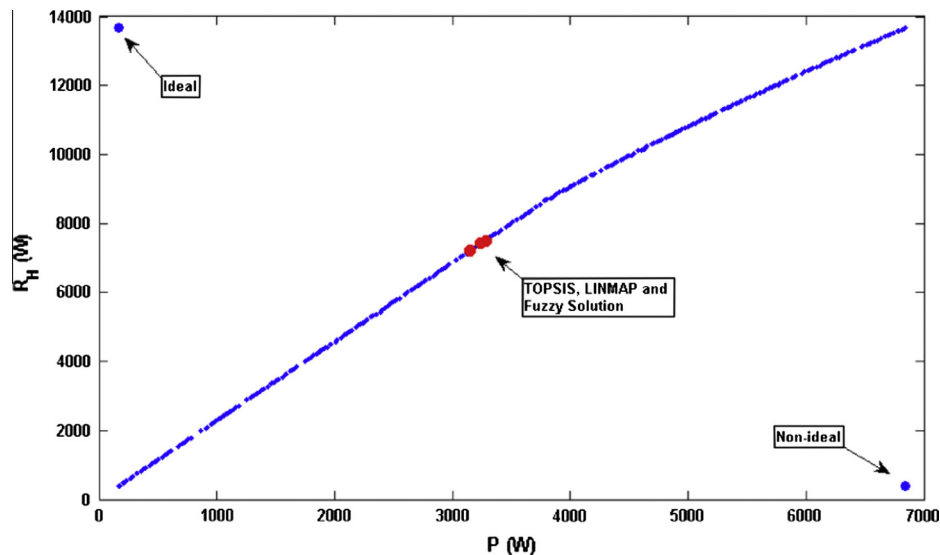


Fig. 4. Pareto frontier (Pareto optimal solutions) for the input power versus heating load using NSGA-II.

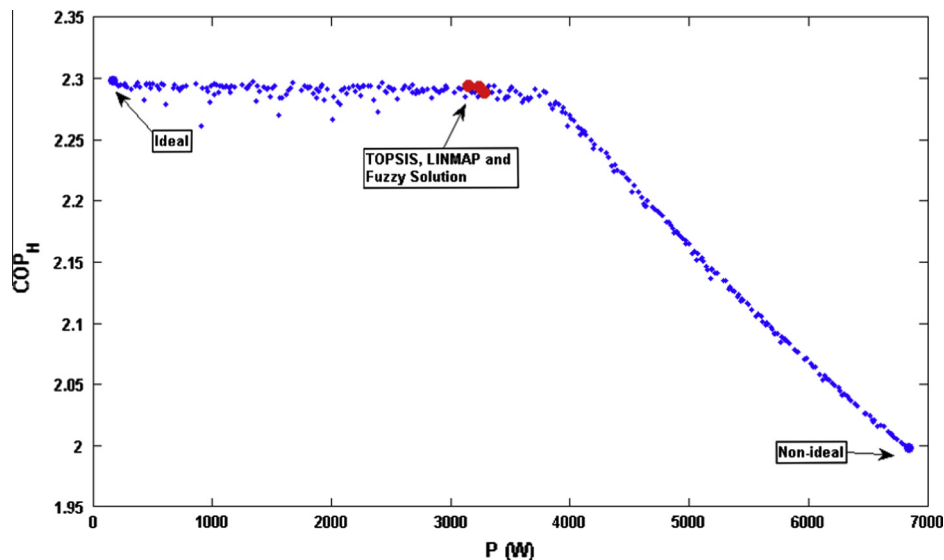


Fig. 5. Pareto frontier (Pareto optimal solutions) for the input power versus the coefficient of performance using NSGA-II.

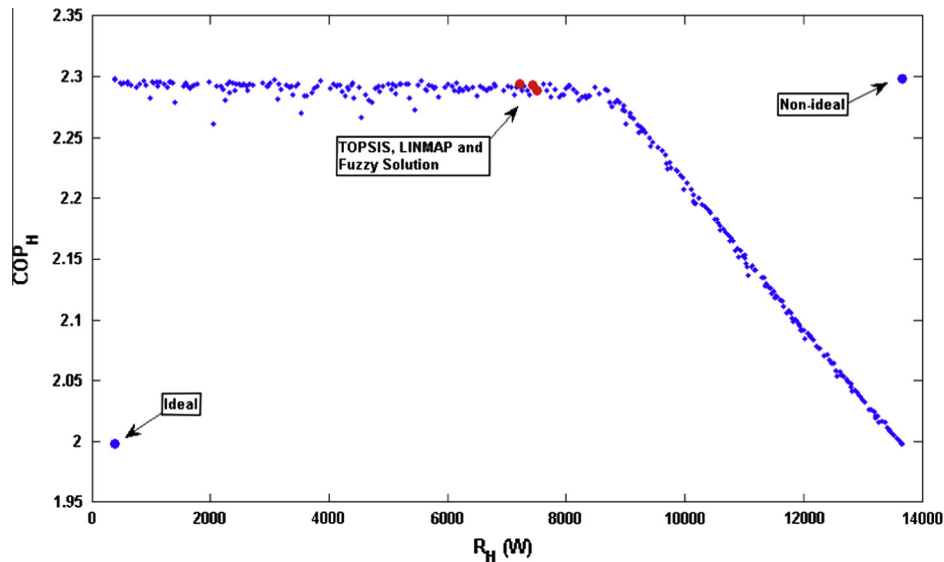


Fig. 6. Pareto frontier (Pareto optimal solutions) for the heating load versus the coefficient of performance using NSGA-II.

from 3000–3500 W and optimal value of COP_H was approximately 2.3.

Pareto optimal frontier for heating load and the coefficient of performance are shown in Fig. 6 that contain two addressed objective functions. The selected data points due to the decision making approaches are illustrated, too. It is seen that optimal value of R_H was 6000–8000 W and optimal value of COP_H was approximately 2.3.

Moreover, the dispensation of different values of design factors throughout their feasible interval for the optimal design data points on the Pareto optimal front are demonstrated in Figs. 7–12. These plots can provide better vision of variation of the design variables from the Pareto frontier.

Fig. 7 depicts that different values of decision variable C_H throughout its entire acceptable range were performed to gain optimal point routes on the Pareto optimal front. In other words, values of C_H , have a wide dispensation throughout the entire Pareto optimal front points as demonstrated in Fig. 7.

Fig. 8 illustrates that different values of decision variable C_L throughout its entire acceptable range were performed to gain optimal point routes on the Pareto optimal front. In other words, values of C_L , have a wide dispensation throughout the entire Pareto optimal front points as demonstrated in Fig. 8. Also, the variation of these addressed parameters for the optimal data points on the Pareto front for ε_H over population are depicted in Fig. 9 whereas upper and lower limit of ε_H referred to 0.5 and 0.8, respectively. The same analogy was employed for ε_L , dedicated to this fact. The assortment of the mentioned variables for the optimal points on Pareto front for the addressed parameter over population is illustrated in Fig. 10 and upper and lower restrictions of ε_L are 0.5 and 0.8, correspondingly.

Fig. 11 depicts the variation of T_c for the optimum points on Pareto front though the lower and upper magnitudes of the referred parameters are marked. Finally, variables distributions for the optimum points on Pareto front for x over population are exhibited

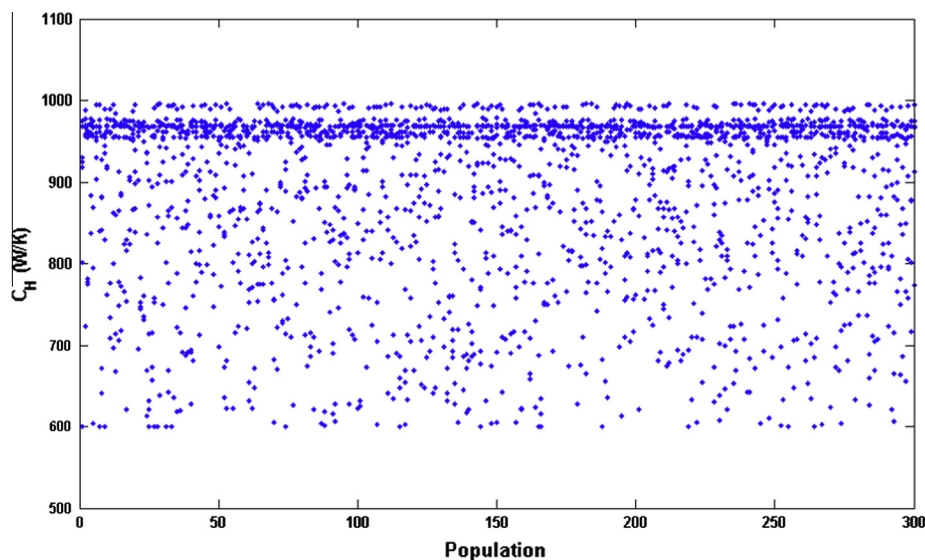


Fig. 7. The distribution of C_H for the optimal points on Pareto front.

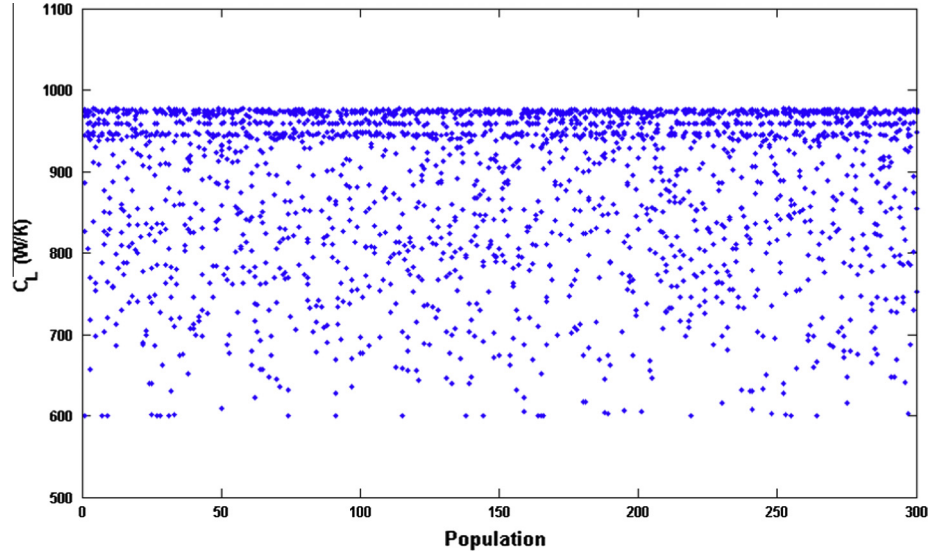


Fig. 8. The distribution of C_L for the optimal points on Pareto front.

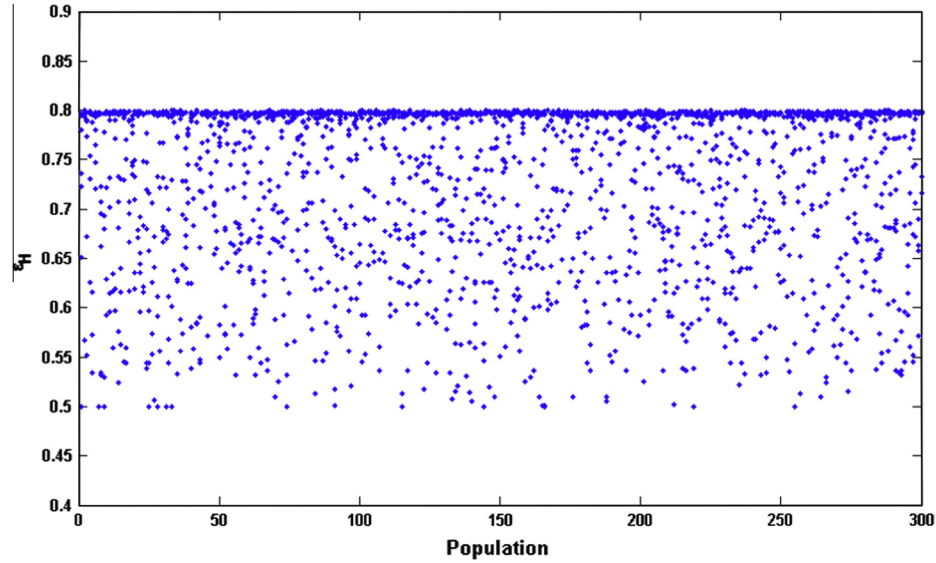


Fig. 9. The distribution of e_H for the optimal points on Pareto front.

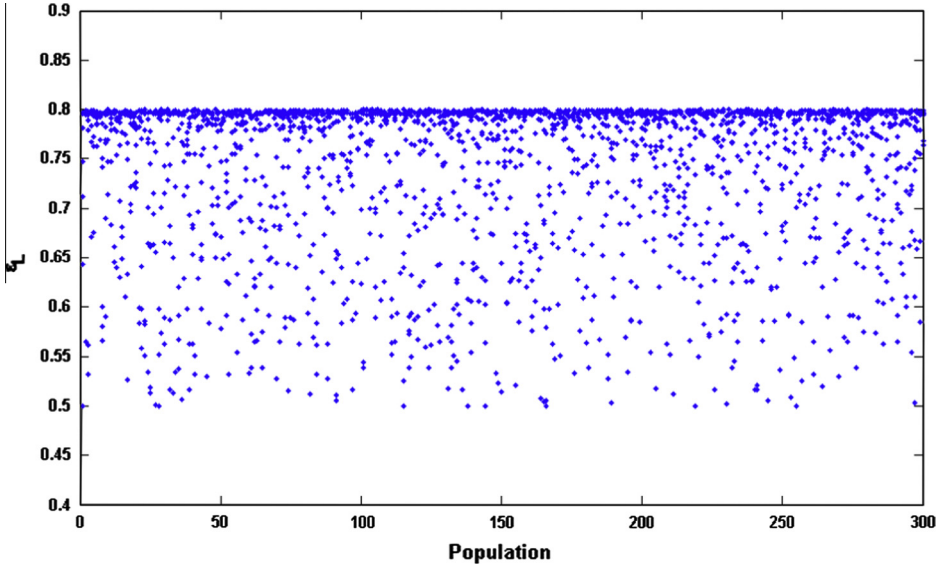


Fig. 10. The distribution of e_L for the optimal points on Pareto front.

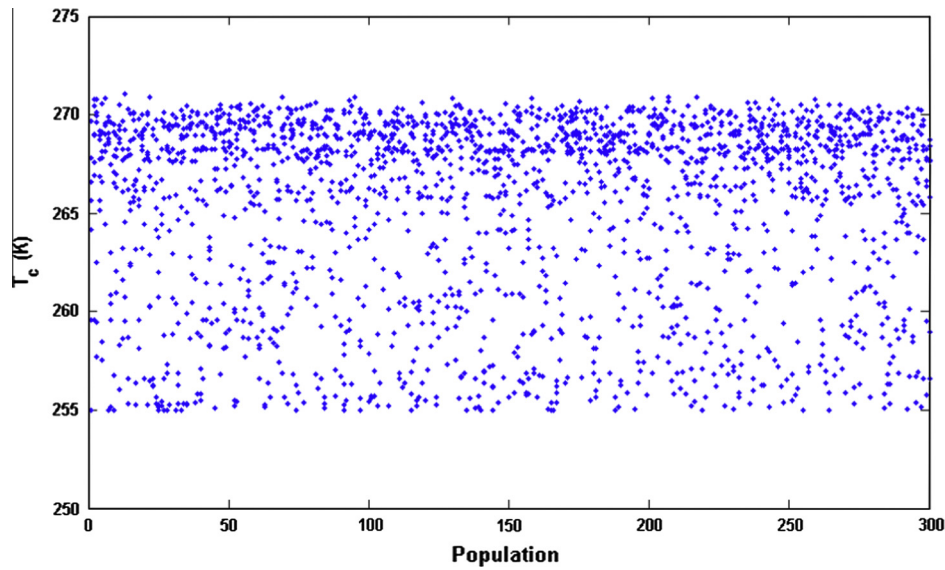


Fig. 11. The distribution of T_c for the optimal points on Pareto front.

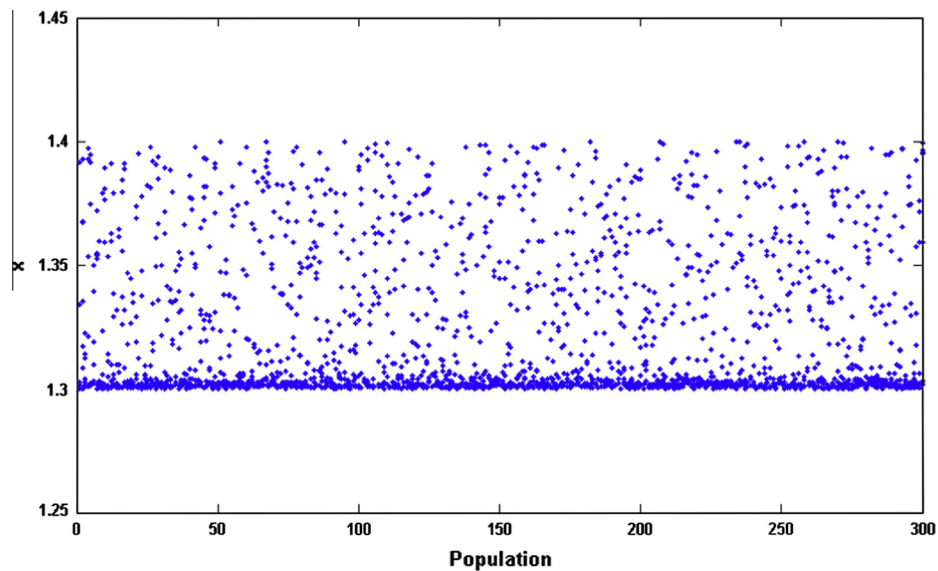


Fig. 12. The distribution of x for the optimal points on Pareto front.

through Fig. 12 and upper and lower ranges of x eluded to 1.3 and 1.4, respectively.

The optimal gained outputs for decision variables and objective functions by utilizing TOPSIS, Bellman–Zadeh, and LINMAP decision making approaches are presented in Table 1.

9. Error analysis

To evaluate precision of each objective at the three decisions making methods, mean absolute percentage error analysis is applied. To calculate the error of each objective, 30 iteration of each

Table 1
Comparison between obtained optimum solutions for three decision making approaches

Decision making method	Decision variables						Objectives		
	e_H	e_L	T_c	x	C_H	C_L	P	R_H	COP_H
TOPSIS	0.725947	0.78958	268.138	1.30118	956.062	798.9327	3147.09	7218.822	2.29381
LINMAP	0.746067	0.75546	267.822	1.30141	939.557	906.0773	3238.60	7426.02	2.29297
Fuzzy	0.742865	0.71707	268.363	1.30257	937.105	927.6334	3279.14	7505.34	2.28882

Table 2

Statistical analysis of the three aforementioned decision making approaches based on MAPE and MAAE

Decision making method	TOPSIS			LINMAP			Fuzzy		
	P	R_H	COP_H	P	R_H	COP_H	P	R_H	COP_H
Max Error%	9.08	8.99	0.16	5.43	5.36	0.09	5.48	5.22	0.31
Average Error%	7.15	7.05	0.11	2.71	2.64	0.06	2.46	2.47	0.18

method were carried out to gain final route by TOPSIS, Fuzzy and LINMAP decision making approaches. First line of Table 2 reports MAAE (Maximum Absolute Percentage Error) of three referred decision making approaches. Besides, second line of Table 2 describes MAPE (Mean Absolute Percentage Error) of abovementioned techniques.

10. Conclusions

In this study, FTT approach has been carried out to indicate input power, the heating load (R_H) and the coefficient of performance (COP_H) of the Stirling heat pump. The input power, the addressed parameters that includes R_H and COP_H of the heat pump have simultaneously been satisfied for multi-objective optimization the performance of the hot-side heat exchanger (ϵ_H), the performance of the cold-side heat exchanger (ϵ_L), rate of the heat capacitance in heat source (C_H), rate of the heat capacitance in heat sink (C_L), the cold side temperature's (T_c) and temperatures ratio (x) have been considered as design variables. Multi objective evolutionary approach that evolved from NSGA-II method has been carried out and the Pareto optimal frontiers throughout the space of objectives were determined accordingly. The best route has been chosen from gained outputs of the Pareto frontier by implementing three robust and fast decision making approaches that includes fuzzy, TOPSIS and LINMAP techniques.

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